

# Thermo-hydraulic design of an unfinned and finned double pipe heat exchanger for milk cooling. Part 1 - Unfinned heat exchanger.

*Diseño térmico-hidráulico de un intercambiador de calor de doble tubo sin y con aletas para el enfriamiento de leche. Parte 1 – Intercambiador de calor sin aletas.*

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Received: 06/06/2025 – Accepted: 29/10/2025 – Published: 01/01/2026

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## Abstract.

Double-pipe heat exchangers (DPHEs) have acquired significance in recent years as a result of their simple construction, compactness, ease of maintenance and cleaning, and relatively low operating/capital costs, with widespread use in heat transfer services involving sensible heating or cooling of process fluids. This paper aims to design a DPHE from the thermo-hydraulic point of view, to determine its suitability and applicability to cool down a stream of liquid cow's milk using chilled water as coolant. Several design parameters were calculated such as total number of hairpins (21), heat transfer surface area (12.92 m<sup>2</sup>), cleanliness factor (0.752) and percent over surface (32.96%), which can be considered as satisfactory. Also, it is required a mass flowrate of chilled water of 9.32 kg/s, classified as high. The designed DPHE cannot be applied satisfactorily in the requested heat transfer service because the pressure drop (9,481,246 Pa) of the tube-side fluid (chilled water) is quite higher than the maximum allowable limit set by the process (85,000 Pa), which also increases the required pumping power for this fluid to an important value (110.5 kW). The designed DPHE will cost around USD \$ 45,600 based on May 2025.

## Keywords.

Unfinned double pipe heat exchanger; thermal design; number of hairpins; pressure drop; pumping power; purchased cost.

## Resumen.

Los intercambiadores de calor de doble tubo (ICDT) han adquirido importancia en años recientes como resultado de su construcción simple, compactación, facilidad de mantenimiento y limpieza, y costos capitales/operación relativamente bajos, con uso extendido en servicios de transferencia de calor que involucren calentamiento y enfriamiento sensible de fluido de proceso. Este artículo tiene como objetivo diseñar un ICDT desde el punto de vista térmico-hidráulico, para determinar su idoneidad y aplicabilidad para enfriar una corriente de leche de vaca líquida usando agua fría como agente de enfriamiento. Varios parámetros de diseño fueron calculados tales como el número total de horquillas (21), área superficial de transferencia de calor (12,92 m<sup>2</sup>), factor de limpieza (0,752) y porcentaje de sobre superficie (32,96%), los cuales pueden considerarse satisfactorios. También, se requiere un caudal máscio de agua fría de 9,32 kg/s, clasificado como elevado. El ICDT diseñado no puede aplicarse satisfactoriamente en el servicio de transferencia de calor demandado debido a que la caída de presión (9 481 246 Pa) del fluido del lado del tubo (agua de enfriamiento) es muy superior que el límite permisible máximo fijado por el proceso (85 000 Pa), lo cual también incrementa la potencia de bombeo requerida para este fluido hacia un valor importante (110,5 kW). El IDCT diseñado costara alrededor de USD \$ 45 600 basado en Mayo del 2025.

## Palabras clave.

Intercambiador de calor de doble tubo; diseño térmico; número de horquillas; caída de presión; costo de adquisición.

## 1. Introduction

Heat exchangers are apparatuses designed to facilitate the transfer of heat between two or more fluids with changing temperatures [1]. In recent decades, the significance of heat exchangers has grown substantially due to their roles in energy efficiency, recovery, and transformation, as well as the integration of alternative energy sources [2].

The thermal energy that passes through a heat exchanger can be either sensible heat or latent heat from the flowing fluids.

The fluid supplying the thermal energy is known as the hot fluid, whereas the fluid that absorbs thermal energy is referred as the cold fluid. Within a heat exchanger, the temperature of the hot fluid is expected to decrease, while the cold fluid's temperature will rise. The primary function of a heat exchanger is to either increase or lower the temperature of the target fluid [3].

Heat exchangers are commonly utilized across various sectors including energy production facilities, chemical

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manufacturing, biotechnology, the food sector, environmental engineering, and the recovery of waste heat, among others. The most basic type of modern heat exchangers is the double pipe heat exchanger [4], which is also referred to as a hairpin heat exchanger [1].

The DPHE was developed in the late 1940s, and research conducted since that time has largely supported its effectiveness for achieving significant developments. This type of heat exchanger facilitates the transfer of thermal energy principally between hot and cold liquids, usually within concentric piping arranged in various arrangements, initially set up in parallel and later adapted to counterflow designs [5].

A DPHE heat exchanger is made up of a one or more tubes arranged concentrically within a larger diameter pipe, featuring fittings designed to direct the flow from one section to another. In this type of heat exchanger, one fluid circulates inside the inner pipe (tube-side), while another fluid moves through the surrounding annular area (annulus). The inner tube is connected via U-shaped bends that are contained in a return-bed housing [1].

A DPHE can be configured in different series and parallel setups [1] to fulfill the needs for pressure drop, heat transfer, and logarithmic mean temperature difference (LMTD) [6].

This type of heat exchanger is utilized for applications involving low flow rates, a wide range of temperatures [7], and high pressure services due to the narrower pipe sizes [1], and is suitable for continuous operations that require low to medium heat duties [8], specifically for processes needing sensible heating or cooling in fluids, where compact small heat transfer surfaces of up to 50 m<sup>2</sup> are necessary [1].

It finds extensive application in typical industries such as food production, chemicals, biotechnology, and gas and oil processes [9], which often require heating or cooling of process fluids, while it is also widely employed in research facilities related to energy engineering [10].

As noted in [7], the DPHE is crucial for tasks like reheating, pasteurization, heating and preheating. Its affordability in terms of design and maintenance makes it a preferred choice, especially for small-scale industries.

As stated in [6], DPHE is a cost-effective option for closed loop cooling systems where a sufficient supply of appropriate water is accessible at an affordable rate to fulfill the thermal needs.

These heat exchangers are suitable for processes where one of the streams is either a gas or a thick liquid, or when the volume is limited under high fouling situations. This is due to their simple cleaning and maintenance processes. They can serve as a substitute for shell-and-tube heat exchangers when operating as a true counter-flow heat exchanger. DPHEs feature an outer pipe ranging from 50 to 400 mm of

internal diameter, and have a standard length of 1.5 to 12.0 m per hairpin. The inner tube's outer diameter can vary from 19 mm to 100 mm. A significant drawback is their bulkiness and high cost per unit of heat transfer surface area [1].

An advantage of the DPHE lies in its affordability in terms of design and maintenance, characterized by a basic configuration that is easy to install, clean, maintain, and adapt, which significantly extends its lifespan and functionality [10].

Peccini et al., [11] suggested that when a stream includes suspended particles, DPHE might be a preferable option since they can incorporate a larger diameter inner tube to prevent blockages. Additionally, this heat exchanger type offers versatility because of its modular design, enabling easier adaptations to modifications in processes. The same authors noted that the longitudinal flow within a DPHE eliminates stagnant zones, which are likely to accumulate deposits in shell and tube exchangers.

It is essential to thermally design heat exchangers in a way that enhances heat transfer while maintaining the pressure drop of the fluids within acceptable limits. A frequent challenge faced by industries is efficiently extracting heat from a utility stream coming out of a specific process and using that energy to heat another process stream.

One way to maximize heat extraction might involve augmenting the heat transfer area or increasing the coolant flow rate; however, both strategies can lead to increased pumping costs, making it unwise to increase these parameters without considering pressure drops. The conventional approach to designing heat exchangers requires careful assessment of all design factors through a detailed process of trial and error, accounting for all potential variations [12].

In [7] it is indicated that engineers encounter significant difficulties while designing an effective heat exchanger. This challenge arises not just from the need to accurately evaluate long-term efficiency and related financial costs, but also from the crucial necessity of thoroughly examining aspects such as heat transfer, pressure drop, and overall effectiveness, which require intensive effort.

According to [13], optimization in the design of heat exchangers is a subject that has been widely explored in existing literature. Most research that has addressed this issue utilized closed-form analytical methods to represent the operational characteristics of the systems, including techniques like the LMTD and effectiveness ( $\epsilon$ -NTU) approaches. Such analytical methods rely on the assumption of consistent physical property values and heat transfer coefficients, which can lead to significant inconsistencies in various design scenarios.

In the design of a DPHE, the majority of academic sources [14,15,16] typically incorporate a broader collection of

design elements, such as physical dimensions, fluid distributions, and configurations involving multiple units. They often rely on a conventional process of experimentation and validation; in this method. The design elements are determined initially, and subsequently, the number of required hairpins for that setup is computed. If the heat exchanger obtained is considered unsuitable—due to reasons like the allowable pressure drop for the specified flow rates falling outside predetermined limits or the streams velocity are not within the required limits—a modification in the design is suggested, and calculations are reconsidered.

This methodology relies heavily on the designer's expertise and does not ensure optimal results. Choices available to designers for new tests are various; they might modify lengths, diameters of pipes, arrangements of hairpins, and other characteristics to achieve a decrease in pressure drop or enhance the heat transfer coefficient. Professionals rely on their intuitive judgments to ultimately develop a viable heat exchanger, which is the primary objective of the design approach [11].

Numerous investigations are reported where a DPHE was designed utilizing different methodologies and tools. In this regard, a comprehensive theoretical and practical study was conducted in [6] where simulations were executed to evaluate the design and functionality of a DPHE. This performance assessment was carried out using computational fluid dynamics (CFD), while the overall effectiveness was also calculated.

Likewise, [9] conducted numerical analysis on how the ratio of pipe diameters and the ratio of diameter to length influence the performance of heat exchangers in a DPHE, utilizing CFD software to model the scenarios with incompressible air. They statistically identified and optimized the factors that lead to the maximum heat transfer under constant flow conditions based on the findings. The researchers noted that their results will aid in future investigations into the design of heat exchangers with optimal dimensions for length and diameter.

Additionally, [13] discussed the use of an integer linear programming (ILP) formulation for designing DPHE. The model used for the heat exchanger relied on discretizing conservation equations; consequently, the physical properties were assessed locally, incorporating their temperature dependence into the model. The numerical findings demonstrated the effectiveness of this proposed method, revealing that analytical methods might either underestimate or overestimate the necessary size of a heat exchanger.

In a similar manner, [8] executed an extensive design and assembly of a laboratory-type DPHE suitable for both parallel and counterflow arrangements. The heat exchanger developed in this research was constructed from galvanized steel for both its tube and shell, while the performance metrics (such as LMTD, heat transfer rate, effectiveness, and

overall heat transfer coefficient) were collected and compared across the two configurations utilized.

In [1], several DPHEs were thermally designed in order to be utilized as oil coolers in naval ships, while the designed DPHE were evaluated with each other regarding the quantity of hairpins, the pressure drop, and the power required for pumping. This assessment incorporated the Nusselt numbers suggested by various researchers across four different design categories: clean finned, fouled finned, clean unfinned, and fouled unfinned.

Similarly, in [10] the effectiveness of existing theoretical approaches for designing a DPHE with narrow tube spacing and low fluid velocities was assessed, corresponding to laminar flow characteristics of the heat transfer fluid within the annulus. This research scrutinized the reasons behind discrepancies when comparing theoretical findings with experimental data, offering suggestions for the proper design of DPHE.

Likewise, in [17] a DPHE was conceptualized, built, and incorporated into an operating biomass gasification facility to capture heat from the syngas released by the gasifier, which has an exit temperature near 350 °C.

In [11], the optimization of a DPHE using mathematical methods was explored, focusing on minimizing the exchanger area while accounting for the thermo-fluid dynamic conditions to apply the appropriate transport correlations, alongside design constraints like maximum pressure drops and minimum excess area.

This research introduced two mixed-integer nonlinear programming strategies, expanding the range of design variables compared to previous studies. These variables included the distribution of fluid streams (either within the inner tube or the annulus), the diameters of both tubes, tube length, the quantity of parallel branches, the number of units arranged in series and parallel within each branch, as well as the number of hairpins in each unit, which affect how the hairpins are configured.

In [12], the most effective design of a DPHE was expressed as a single-variable geometric programming challenge. Solving this issue provides the optimal dimensions for the inner and outer pipe diameters and the utility flowrate necessary for a DPHE of a specified length, given a predetermined flowrate for the process stream and a defined temperature difference from inlet to outlet.

In [18], a DPHE was designed to investigate the heat transfer process occurring between two fluids (water/water) through a solid separator. It was developed with a counterflow setup, utilizing the LMTD analysis method.

In [19], a method combining gray relational analysis (GRA) with artificial neural networks (ANNs) and genetic algorithms (GAs) was utilized to assess the importance of

parameters such as effectiveness, thermal resistance, and overall heat transfer coefficient, to rank these parameters in a specific sequence. The integrated methodology introduced in this research has the potential to enhance problem-solving abilities and offer insightful knowledge to improve heat exchanger performance across different industries.

In [20] the calculation of thermal design parameters of a DPHE was outlined to ensure effective heating and sterilization of an organic fluid stream used in the seed-skin separation process for various vegetables.

Lastly, [21] explored both analytical and numerical methods in designing a DPHE. The analysis included the consideration of sensible heat transfer, and the heat exchanger was customized to fit the real operating conditions of a chemical facility. This research employed an analytical model using the effectiveness-number of transfer units ( $\epsilon$ -NTU) method alongside the LMTD approach in the design of the DPHE, with performance charts created during the design phase for the specified heat exchanger.

In a Cuban dairy processing plant it's required to cold down a liquid cow's milk stream using chilled water, and for that two DPHEs have been proposed, the first one unfinned and the second with longitudinal fins in the inner tube (finned). In this context, the present paper is the first part of a two-part project, where an unfinned DPHE is designed in order to know if this heat exchanger is suitable to implement in this heat transfer service through the calculation of several design parameters such as the total number of hairpins, the cleanliness factor, the percent over surface, the pressure drop and the pumping power of both liquid streams, among others.

Likewise, the purchase cost of the unfinned DPHE was also calculated. In the second paper, a finned DPHE is designed where the key design parameters previously mentioned are also computed, while the results will be compared and evaluated with respect to those calculated for the unfinned DPHE of the present study, in order to select the most suitable, economical and applicable DPHE from the thermo-hydraulic point of view to carry out this heat transfer service.

## 2. Materials and methods.

### 2.1. Problem statement.

It is required to cool down 4,320 kg/h of a liquid cow's milk stream from 60 °C to 10 °C by means of chilled water available at 2 °C, where the outlet temperature of the chilled water stream must not exceed 8 °C. The following data are available for the tube and the annulus:

- Nominal diameter annulus: 2 in.
- Nominal diameter inner tube: 1 in.
- Length of tube: 3 m.
- Number of tubes inside the annulus: 1.
- Tube material: Carbon steel.
- Thermal conductivity of the tube material: 52 W/m.K.

Design an unfinned double pipe heat exchanger using the methodology reported by [15], where several thermo-hydraulic and design parameters should be calculated such as the heat transfer surface area, the total number of hairpins, the cleanliness factor, the percent over surface, the pressure drop and the pumping power of both liquid streams. It's required that the pressure drop for both the tube-side and annulus fluid don't exceed 85,000 Pa. Lastly; calculate the purchased equipment cost of the designed DPHE and update it to 2025.

### 2.2. Design methodology.

#### Percent over surface

Step 1. Definition of the initial parameters for the streams: Table 1 presents the initial parameters that must be defined for both fluid streams

Table 1. Initial parameters to be defined for both streams.

Parameter	Hot fluid	Cold fluid	Units
Mass flowrate	$m_h$	$m_c$	kg/h
Inlet temperature	$T_1$	$t_1$	°C
Outlet temperature	$T_2$	$t_2$	°C
Maximum allowable pressure drop	$\Delta P_{mh}$	$\Delta P_{mc}$	Pa
Fouling factor	$R_h$	$R_c$	$m^2.K/W$

Source: Own elaboration.

Step 2. Definition of the geometric dimensions for the hairpins:

Table 2 shows the geometric dimensions to be defined for the hairpins.

Table 2. Geometric dimensions to be defined for the hairpins.

Parameter	Symbol	Units
Tube length	$L_t$	m
Internal diameter annulus	$D_i$	m
Internal diameter inner tube	$d_i$	m
External diameter inner tube	$d_e$	m
Thermal conductivity metallic material of the inner pipe	$k_m$	W/m.K

Source: Own elaboration.

Step 3. Definition of the flow arrangement inside the double-pipe heat exchanger:

- Counterflow.
- Parallel.

Step 4. Allocation of fluids inside the double-pipe heat exchanger

Step 5. Consider insulation of the double-pipe heat exchanger against heat losses.

Step 6. Average temperature of both fluids:

- Hot fluid ( $T$ ):

$$\bar{T} = \frac{T_1 + T_2}{2} \quad (1.1)$$

- Cold fluid ( $\bar{t}$ ):

$$\bar{t} = \frac{t_1 + t_2}{2} \quad (1.2)$$

Step 7. Physical parameters of both fluids at the average temperature:

Table 3 displays the physical properties that must be defined for both fluids at the average temperature calculated in the previous step.

Table 3. Physical parameters to be defined for both fluids.

Parameter	Hot fluid	Cold fluid	Units
Density	$\rho_h$	$\rho_c$	kg/m <sup>3</sup>
Viscosity	$\mu_h$	$\mu_c$	Pa.s
Heat capacity	$Cp_h$	$Cp_c$	kJ/kg.K
Thermal conductivity	$k_h$	$k_c$	W/m.K

Source: Own elaboration.

Step 8. Heat load ( $Q$ ):

- Using the data for the hot fluid:

$$Q = \frac{m_h}{3,600} \cdot Cp_h \cdot (T_1 - T_2) \quad (1.3)$$

- Using the data for the cold fluid:

$$Q = \frac{m_c}{3,600} \cdot Cp_c \cdot (t_2 - t_1) \quad (1.4)$$

Where both  $m_h$  and  $m_c$  are given in kg/h.

Step 9. Mass flowrate of one stream:

- Mass flowrate of the hot fluid:

$$m_h = \frac{Q}{Cp_h \cdot (T_1 - T_2)} \quad (1.5)$$

- Mass flowrate of the cold fluid:

$$m_c = \frac{Q}{Cp_c \cdot (t_2 - t_1)} \quad (1.6)$$

Step 10. Tube wall temperature ( $T_w$ ):

$$T_w = \frac{1}{2} \cdot (\bar{T} + \bar{t}) \quad (1.7)$$

Step 11. Viscosity of both fluids at the tube wall temperature:

Hot fluid ( $\mu_{hw}$ ) [Pa.s].

Cold fluid ( $\mu_{cw}$ ) [Pa.s].

Step 12. Net free flow area of the inner tube ( $A_{ct}$ ):

$$A_{ct} = \frac{\pi \cdot d_i^2}{4} \quad (1.8)$$

Step 13. Velocity of the tube-side fluid ( $v_t$ ):

$$v_t = \frac{m_t}{\rho_t \cdot A_{ct}} \quad (1.9)$$

Where  $m_t$  is given in kg/s.

Step 14. Reynolds number of the tube-side fluid ( $Re_t$ ):

$$Re_t = \frac{\rho_t \cdot v_t \cdot d_i}{\mu_t} \quad (1.10)$$

Step 15. Prandtl number of the tube-side fluid ( $Pr_t$ ):

$$Pr_t = \frac{Cp_t \cdot \mu_t}{k_t} \quad (1.11)$$

Where  $Cp_t$  is given in J/kg.K.

Step 16. Nusselt number for the tube-side fluid ( $Nu_t$ ):

- Laminar flow ( $Re_t < 2,300$ )

$$Nu_t = 1.86 \cdot (Re_t \cdot Pr_t)^{0.33} \cdot \left( \frac{d_i}{L_t} \right)^{0.33} \cdot \left( \frac{\mu_t}{\mu_{tw}} \right)^{0.14} \quad (1.12)$$

Valid for smooth tubes for the following conditions:

$$0.48 < Re_t \cdot Pr_t < 16,700$$

$$0.0044 < \left( \frac{\mu_t}{\mu_{tw}} \right)^{0.14} < 9.75$$

$$(Re_t \cdot Pr_t \cdot \frac{d_i}{L_t})^{0.33} \left( \frac{\mu_t}{\mu_{tw}} \right)^{0.14} \geq 2$$

- Turbulent flow ( $2,300 < Re_t < 10^4$ ) [Gnielinski's correlation]:

$$Nu_t = \frac{\left( \frac{f_t}{2} \right) \cdot (Re_t - 1,000) \cdot Pr_t}{1 + 12.7 \cdot \left( \frac{f_t}{2} \right)^{0.5} \cdot (Pr_t^{2/3} - 1)} \quad (1.13)$$

Where  $f_t$  is the Fanning friction factor for the tube-side fluid and is calculated using the following correlation:

$$f_t = (1.58 \cdot \ln Re_t - 3.28)^{-2} \quad (1.14)$$

- Turbulent flow ( $10^4 < Re_t < 5 \times 10^6$ ) [Prandtl's correlation]:

$$Nu_t = \frac{\left( \frac{f_t}{2} \right) \cdot Re_t \cdot Pr_t}{1 + 8.7 \cdot \left( \frac{f_t}{2} \right)^{0.5} \cdot (Pr_t - 1)} \quad (1.15)$$

Valid for  $Pr_t > 0.5$ .

Where:

$$f_t = (1.58 \cdot \ln Re_t - 3.28)^{-2} \quad (1.14)$$

Step 17. Heat transfer coefficient for the tube-side fluid ( $h_t$ ):

$$h_t = \frac{Nu_t \cdot k_t}{d_i} \quad (1.16)$$

Step 18. Net free flow area of the annulus ( $A_{ca}$ ):

$$A_{ca} = \frac{\pi \cdot (D_i^2 - d_e^2)}{4} \quad (1.17)$$

Step 19. Velocity of the annulus fluid ( $v_a$ ):

$$v_a = \frac{\frac{m_a}{3,600}}{\rho_a \cdot A_{ca}} \quad (1.18)$$

Step 20. Hydraulic diameter ( $D_h$ ):

$$D_h = D_i - d_e \quad (1.19)$$

Step 21. Reynolds number of the annulus fluid ( $Re_a$ ):

$$Re_a = \frac{\rho_a \cdot v_a \cdot D_h}{\mu_a} \quad (1.20)$$

Step 22. Prandtl number of the annulus fluid ( $Pr_a$ ):

$$Pr_a = \frac{Cp_a \cdot \mu_a}{k_a} \quad (1.21)$$

Where  $Cp_a$  is given in J/kg.K.

Step 23. Nusselt number for the annulus fluid ( $Nu_a$ ):

- Laminar flow ( $Re_a < 2,300$ )

$$Nu_a = 1.86 \cdot (Re_a \cdot Pr_a)^{0.33} \cdot \left( \frac{D_h}{L_t} \right)^{0.33} \cdot \left( \frac{\mu_a}{\mu_{aw}} \right)^{0.14} \quad (1.22)$$

Valid for smooth tubes for the following conditions:

$$0.48 < Re_a \cdot Pr_a < 16,700$$

$$0.0044 < \left( \frac{\mu_a}{\mu_{aw}} \right)^{0.14} < 9.75$$

$$(Re_a \cdot Pr_a \cdot \frac{D_h}{L_t})^{0.33} \left( \frac{\mu_a}{\mu_{aw}} \right)^{0.14} \geq 2$$

- Turbulent flow ( $2,300 < Re_a < 10^4$ ) [Gnielinski's correlation]:

$$Nu_a = \frac{\left( \frac{f_a}{2} \right) \cdot (Re_a - 1,000) \cdot Pr_a}{1 + 12.7 \cdot \left( \frac{f_a}{2} \right)^{0.5} \cdot (Pr_a^{2/3} - 1)} \quad (1.23)$$

Where  $f_a$  is the Fanning friction factor for the annulus fluid and is calculated using the following correlation:

$$f_a = (1.58 \cdot \ln Re_a - 3.28)^{-2} \quad (1.24)$$

- Turbulent flow ( $10^4 < Re_a < 5 \times 10^6$ ) [Prandtl's correlation]:

$$Nu_a = \frac{\left( \frac{f_a}{2} \right) \cdot Re_a \cdot Pr_a}{1 + 8.7 \cdot \left( \frac{f_a}{2} \right)^{0.5} \cdot (Pr_a - 1)} \quad (1.25)$$

Valid for  $Pr_a > 0.5$ .

Where:

$$f_a = (1.58 \cdot \ln Re_a - 3.28)^{-2} \quad (1.24)$$

Step 24. Equivalent diameter for heat transfer ( $D_e$ ):

$$D_e = \frac{D_i^2 - d_e^2}{d_e} \quad (1.26)$$

Step 25. Heat transfer coefficient for the annulus fluid ( $h_a$ ):

$$h_a = \frac{Nu_a \cdot k_a}{D_e} \quad (1.27)$$

Step 26. Fouled overall heat transfer coefficient based on the outside area of the inner tube ( $U_f$ ):

$$U_f = \frac{1}{\frac{d_e}{d_i \cdot h_t} + \frac{d_e \cdot R_t}{d_i} + \frac{d_e \cdot \ln \left( \frac{d_e}{d_i} \right)}{2 \cdot k_m} + R_a + \frac{1}{h_a}} \quad (1.28)$$

Step 27. Log-mean temperature difference ( $\Delta T_m$ ):

- For parallel flow:

$$\Delta T_m = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln \frac{(T_1 - t_1)}{(T_2 - t_2)}} \quad (1.29)$$

- For counterflow:

$$\Delta T_m = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln \frac{(T_1 - t_2)}{(T_2 - t_1)}} \quad (1.30)$$

Step 28. Heat transfer surface area ( $A_o$ ):

$$A_o = \frac{Q \cdot 1,000}{U_f \cdot \Delta T_m} \quad (1.31)$$

Where  $Q$  is given in kW.

Step 29. Heat transfer area per hairpin ( $A_{hp}$ ):

$$A_{hp} = 2 \cdot \pi \cdot d_e \cdot L_t \quad (1.32)$$

Step 30. Number of hairpins ( $N_h$ ):

$$N_h = \frac{A_o}{A_{hp}} \quad (1.33)$$

Step 31. Clean overall heat transfer coefficient based on the outside heat transfer area ( $U_c$ ):

$$U_c = \frac{1}{\frac{d_e}{d_i \cdot h_t} + \frac{d_e \cdot \ln \left( \frac{d_e}{d_i} \right)}{2 \cdot k_m} + \frac{1}{h_a}} \quad (1.34)$$

Step 32. Cleanliness factor ( $CF$ ):

$$CF = \frac{U_f}{U_c} \quad (1.35)$$

Step 33. Total fouling ( $R_{ft}$ ):

$$R_{ft} = \frac{1 - CF}{U_c \cdot CF} \quad (1.36)$$

Step 34. Percent over surface (OS):

$$OS = 100 \cdot U_c \cdot R_{ft} \quad (1.37)$$

*Pressure drop and pumping power*

Step 35. Frictional pressure drop of the tube-side fluid ( $\Delta p_t$ ):

$$\Delta p_t = 4 \cdot f_t \cdot \frac{2 \cdot L_t}{d_i} \cdot N_h \cdot \frac{\rho_t \cdot v_t^2}{2} \quad (1.38)$$

Where for laminar flow ( $Re_t < 2,300$ ):

$$f_t = \frac{16}{Re_t} \quad (1.39)$$

Correction of the Fanning friction factor for laminar flow ( $f_{ct}$ ):

$$f_{ct} = f_t \cdot \left( \frac{\mu_t}{\mu_{tw}} \right)^m \quad (1.40)$$

Where  $m = -0.58$  for heating and  $-0.50$  for cooling under laminar flow.

Step 36. Pumping power for the tube-side fluid ( $P_t$ ):

$$P_t = \frac{\Delta p_t \cdot m_t}{\eta_p \cdot \rho_t} \quad (1.41)$$

Where  $m_t$  is given in kg/s and  $\eta_p$  is the isentropic efficiency of the pump.

Step 37. Frictional pressure drop of the annulus fluid ( $\Delta p_a$ ):

$$\Delta p_a = 4 \cdot f_a \cdot \frac{2 \cdot L_a}{D_h} \cdot \rho_a \cdot \frac{v_a^2}{2} \cdot N_h \quad (1.42)$$

Where for laminar flow ( $Re_a < 2,300$ ):

$$f_a = \frac{16}{Re_a} \quad (1.43)$$

Correction of the Fanning friction factor for laminar flow ( $f_{ca}$ ):

$$f_{ca} = f_a \cdot \left( \frac{\mu_a}{\mu_{aw}} \right)^m \quad (1.44)$$

Where  $m = -0.58$  for heating and  $-0.50$  for cooling under laminar flow.

Step 38. Pumping power for the annulus fluid ( $P_a$ ):

$$P_a = \frac{\Delta p_a \cdot m_a}{\eta_p \cdot \rho_a} \quad (1.45)$$

Where  $m_a$  is given in kg/s and  $\eta_p$  is the isentropic efficiency of the pump.

*Purchased equipment cost*

According to [22], the purchased equipment cost for a DPHE is calculated using the following correlation:

$$C_{DPHE}^{2007} = 1,600 + 2,100 \cdot A_o^{1.0} \quad (1.46)$$

Where:

- $C_{DPHE}^{2007}$  - Purchased equipment cost of the DPHE referred to January 2007 (USD \$).
- $A_o$  - Heat transfer surface area of the DPHE, calculated in Step 28 ( $m^2$ ).

Later on, this purchased equipment cost calculated by equation (1.46) is updated to March 2025 using the Chemical Engineering Plant Cost Index corresponding to March 2025 and by applying the following equation:

$$C_{DPHE}^{2025} = C_{DPHE}^{2007} \cdot \frac{CEPCI^{2025}}{CEPCI^{2007}} \quad (1.47)$$

Where:

- $C_{DPHE}^{2025}$  - Purchased equipment cost of the DPHE referred to May 2025 (USD \$).
- $C_{DPHE}^{2007}$  - Purchased equipment cost of the DPHE based to January 2007 (USD \$).
- $CEPCI^{2025}$  - Chemical Engineering Plant Cost Index referred to May 2025 = 806.8 [23].
- $CEPCI^{2007}$  - Chemical Engineering Plant Cost Index referred to January 2007 = 509.7 [22].

### 3. Analysis and Interpretation of Results.

#### 3.1. Percent over surface.

Step 1. Definition of the initial parameters for the streams: The following table (Table 4) presents the values of the initial parameters to be defined for both streams.

Table 4. Values of the initial parameters to be defined for both streams.

Parameter	Hot fluid (Milk)		Cold fluid (Water)		Units
	Symb ol	Value	Symb ol	Value	
Mass flowrate	$m_h$	4,320	$m_c$	-	kg/h
Inlet temperatur e	$T_1$	60	$t_1$	2	°C
Outlet temperatur e	$T_2$	10	$t_2$	8	°C
Maximum allowable pressure drop	$\Delta P_{mh}$	85,00 0	$\Delta P_{mc}$	85,000	Pa

Fouling factor	$R_h$	0.0001 <sup>δ</sup>	$R_c$	0.00017 <sup>φ</sup>	$m^2 \cdot K / W$
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Source: Own elaboration.

<sup>δ</sup> Taken from [14].

<sup>φ</sup> Taken from [15].

Step 2. Definition of the geometric dimensions for the hairpins:

Table 5 shows the values of the geometric dimensions to be defined for the hairpins.

Table 5. Values of the geometric dimensions to be defined for the hairpins.

Parameter	Symbol	Value	Units
Length	$L_t$	3	m
Internal diameter annulus	$D_i$	0.05250 <sup>φ</sup>	m
Internal diameter inner tube	$d_i$	0.02664 <sup>φ</sup>	m
External diameter inner tube	$d_e$	0.03340 <sup>φ</sup>	m
Thermal conductivity metallic material of the inner pipe	$k_m$	52	W/m.K

Source: Own elaboration.

<sup>φ</sup> According to [15].

Step 3. Definition of the flow arrangement inside the double-pipe heat exchanger:

The fluids will flow under counterflow arrangement inside the DPHE.

Step 4. Allocation of fluids inside the double-pipe heat exchanger.

As suggested by [14] and [22], the hot fluid (milk) will be located in the annulus, while the cold fluid (water) will be located in the inner pipe.

Step 5. Consider insulation of the double-pipe heat exchanger against heat losses.

The heat exchanger will be thermally insulated to avoid excessive heat losses.

Step 6. Average temperature of both streams:

- Hot fluid (milk) ( $T$ ):

$$\bar{T} = \frac{T_1 + T_2}{2} = \frac{60 + 10}{2} = 35 \text{ } ^\circ\text{C} \quad (1.1)$$

- Cold fluid (water) ( $\bar{t}$ ):

$$\bar{t} = \frac{t_1 + t_2}{2} = \frac{2 + 8}{2} = 5 \text{ } ^\circ\text{C} \quad (1.2)$$

Step 7. Physical parameters of both fluids at the average temperature:

According to [24,25,26], both the milk and the water have the physical parameters presented in Table 6 at the average temperature calculated in the previous step.

Table 6. Values of the physical parameters for the milk and the water.

Parameter	Hot fluid (Milk)		Cold fluid (Water)		Units
	Symb ol	Value	Symb ol	Value	
Density	$\rho_h$	1,013. 2	$\rho_c$	999.9 7	kg/m <sup>3</sup>
Viscosity	$\mu_h$	0.001 06	$\mu_c$	0.001 52	Pa.s
Heat capacity	$Cp_h$	3.919	$Cp_c$	4.205	kJ/kg. K
Thermal conductivity	$k_h$	0.580	$k_c$	0.571	W/m. K

Source: Own elaboration.

Step 8. Heat load ( $Q$ ):

- Using the data for the hot fluid (milk):

$$Q = \frac{m_h}{3,600} \cdot Cp_h \cdot (T_1 - T_2) \quad (1.3)$$

$$Q = \frac{4,320}{3,600} \cdot 3.919 \cdot (60 - 10) = 235.14 \text{ kW}$$

Where  $m_h$  is given in kg/h.

Step 9. Mass flowrate of one stream:

- Mass flowrate of the cold fluid (water):

$$m_c = \frac{Q}{Cp_c \cdot (t_2 - t_1)} = \frac{235.14}{4.205 \cdot (8 - 2)} = 9.32 \text{ kg/s} \quad (1.6)$$

Step 10. Tube wall temperature ( $T_w$ ):

$$T_w = \frac{1}{2} \cdot (\bar{T} + \bar{t}) = \frac{1}{2} \cdot (35 + 5) = 20 \text{ } ^\circ\text{C} \quad (1.7)$$

Step 11. Viscosity of both fluids at the tube wall temperature: According to [25,26], both the milk and the water present the following values of the viscosity at  $T_w = 20 \text{ } ^\circ\text{C}$ .

- Hot fluid (milk) ( $\mu_{hw}$ ) [Pa.s] = 0.00205 Pa.s
- Cold fluid ( $\mu_{cw}$ ) [Pa.s] = 0.00100 Pa.s

Step 12. Net free flow area of the inner tube ( $A_{ct}$ ):

$$A_{ct} = \frac{\pi \cdot d_i^2}{4} = \frac{\pi \cdot 0.02664^2}{4} = 0.00056 \text{ m}^2 \quad (1.8)$$

Because the cold fluid (water) will flow in the inner tube, and the hot fluid (milk) will flow in the annulus, the following new nomenclature presented in Table 7 will be applied for the flowrates, physical parameters and fouling factors of both streams.

Table 7. New nomenclature to be applied for both streams.

Hot fluid (milk)	Cold fluid (water)
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Parameter	Former nomenclature	New nomenclature	Former nomenclature	New nomenclature		Reynolds number of the annulus fluid (milk)	$Re_a$	16,796	-
Flowrate	$m_h$	$m_a$	$m_c$	$m_t$					
Density	$\rho_h$	$\rho_a$	$\rho_c$	$\rho_t$	22	Prandtl number of the annulus fluid (milk)	$Pr_a$	7.16	-
Viscosity	$\mu_h$	$\mu_a$	$\mu_c$	$\mu_t$					
Heat capacity	$Cp_h$	$Cp_a$	$Cp_c$	$Cp_t$		Fanning friction factor for the annulus fluid (milk)	$f_a$	0.00684	-
Thermal conductivity	$k_h$	$k_a$	$k_c$	$k_t$	23	Nusselt number for the annulus fluid (milk) <sup>2</sup>	$Nu_a$	99.49	-
Fouling factor	$R_h$	$R_a$	$R_c$	$R_t$		Equivalent diameter for heat transfer	$D_e$	0.0491	m

Source: Own elaboration.

Table 8 displays the values of the parameters included in steps 13-25.

Table 8. Values of the parameters included in steps 13-25.

Step	Parameter	Symbol	Value	Units
13	Velocity of the tube-side fluid (water)	$v_t$	16.64	m/s
14	Reynolds number of the tube-side fluid (water)	$Re_t$	291,629	-
15	Prandtl number of the tube-side fluid (water)	$Pr_t$	11.19	-
16	Fanning friction factor for the tube-side fluid (water)	$f_t$	0.00362	-
17	Nusselt number for the tube-side fluid (water) <sup>1</sup>	$Nu_t$	1,237.84	-
18	Heat transfer coefficient for the tube-side fluid (water)	$h_t$	26,531.78	W/m <sup>2</sup> .K
19	Net free flow area of the annulus	$A_{ca}$	0.00129	m <sup>2</sup>
20	Velocity of the annulus fluid (milk)	$v_a$	0.92	m/s
	Hydraulic diameter	$D_h$	0.0191	m

21	Reynolds number of the annulus fluid (milk)	$Re_a$	16,796	-
22	Prandtl number of the annulus fluid (milk)	$Pr_a$	7.16	-
23	Fanning friction factor for the annulus fluid (milk)	$f_a$	0.00684	-
24	Nusselt number for the annulus fluid (milk) <sup>2</sup>	$Nu_a$	99.49	-
25	Equivalent diameter for heat transfer	$D_e$	0.0491	m
	Heat transfer coefficient for the annulus fluid (milk)	$h_a$	1,175.24	W/m <sup>2</sup> .K

Source: Own elaboration.

<sup>1</sup>Since  $10^4 < Re_t < 5 \times 10^6$ , the tube-side fluid (water) flows under turbulent regime, thus Prandtl's correlation (equation 1.15) was used to calculate the Nusselt number for this fluid. This equation is also valid to use because  $Pr_t = 11.19 > 0.5$ .

<sup>2</sup>Since  $10^4 < Re_a < 5 \times 10^6$ , the annulus side fluid (milk) flows under turbulent regime, thus Prandtl's correlation (equation 1.25) will be used to calculate the Nusselt number for this fluid. This equation is also valid to use because  $Pr_a = 7.16 > 0.5$ .

Table 9 reveals the values of the parameters included in steps 26-34.

Table 9. Values of the parameters included in steps 26-34.

Step	Parameter	Symbol	Value	Units
26	Fouled overall heat transfer coefficient based on the outside area of the inner tube	$U_f$	774.31	W/m <sup>2</sup> .K
27	Log-mean temperature difference <sup>1</sup>	$\Delta T_m$	23.51	°C
28	Heat transfer surface area	$A_o$	12.92	m <sup>2</sup>
29	Heat transfer area per hairpin	$A_{hp}$	0.629	m <sup>2</sup>

30	Number of hairpins	$N_h$	21	-
31	Clean overall heat transfer coefficient based on the outside heat transfer area	$U_c$	1,030.11	W/m <sup>2</sup> .K
32	Cleanliness factor	$CF$	0.752	-
33	Total fouling	$R_{ft}$	0.00032	m <sup>2</sup> .K/W
34	Percent over surface	$OS$	32.96	%

Source: Own elaboration.

<sup>1</sup>For counterflow arrangement.

### 3.2. Pressure drop and pumping power.

Table 10 presents the values of the parameters included in steps 35-38.

Table 10. Values of the parameters included in steps 35-38.

Step	Parameter	Symbol	Value	Units
35	Frictional pressure drop of the tube-side fluid (water)	$\Delta p_t$	9,481,246	Pa
36	Pumping power for the tube-side fluid (water) <sup>1</sup>	$P_t$	110.5	kW
37	Frictional pressure drop of the annulus fluid (milk)	$\Delta p_a$	77,392	Pa
38	Pumping power for the annulus fluid (milk) <sup>1</sup>	$P_a$	114.58	W

Source: Own elaboration.

<sup>1</sup>A value of 0.80 was selected for the isentropic efficiency of the pump [15].

### 3.3. Purchased equipment cost

Using equation (1.46) and for a value of the heat transfer surface area of 12.92 m<sup>2</sup>, the purchased equipment cost of the designed DPHE, based on January 2007, is:

$$\begin{aligned} C_{DPHE}^{2007} &= 1,600 + 2,100 \cdot A_o^{1.0} \\ &= \text{USD } \$ 28,732 \end{aligned} \quad (1.46)$$

$$C_{DPHE}^{2007} \approx 28,800$$

Accordingly, the purchased cost of the designed DPHE, referred to May 2025, is:

$$\begin{aligned} C_{DPHE}^{2025} &= C_{DPHE}^{2007} \cdot \frac{CEPCI^{2025}}{CEPCI^{2007}} = 28,800 \cdot \frac{806.8}{509.7} \\ C_{DPHE}^{2025} &= \text{USD } \$ 45,588 \approx 45,600 \end{aligned} \quad (1.47)$$

### 4. Discussion

The heat load had a relatively high value of 235.14 kW, while it is needed a mass flowrate of 9.32 kg/s for the chilled water, which can be considered high. This is because the low value required for the outlet temperature of the chilled water stream (8 °C) which reduced the cold fluid temperature difference ( $\Delta t = t_2 - t_1 = 6$  °C), whereas the somewhat high value of the liquid milk flowrate (4,320 kg/h or 1.2 kg/s) and the relatively high temperature difference of the milk stream ( $\Delta T = T_1 - T_2 = 50$  °C) both also influence in the relatively high value of the heat load, which in turn effects on the high value obtained for the mass flowrate of chilled water, as shown by equation (1.6). In [15] the value of the heat load was 87.1 kW for a water-to-water DPHE.

The value of the velocity of the tube-side fluid (chilled water) is high (16.64 m/s), which is due to the high value obtained for the chilled water mass flowrate. This value of chilled water velocity is 18 times higher than the calculated value of the velocity (0.92 m/s) for the annulus fluid (milk), and is well above the recommended range reported by [22] for the velocity of water in tubular heat exchangers (1.5-2.5 m/s).

The Reynolds number of the tube-side fluid (chilled water) was 291,629, which is 17.4 times higher than the Reynolds number (16,796) of the annulus fluid (milk). This high value obtained for the Reynolds number of the chilled water stream occurs essentially because the high value of the velocity obtained for this fluid. This result agrees with the unfinned water-to-water DPHE designed in [15], where the value of the Reynolds number of the tube-side fluid (159,343) is higher than the Reynolds number of the annulus fluid (15,201).

In case of the Prandtl number, the value of this parameter for the chilled water (11.19) was 1.56 times higher than the Prandtl number for the milk (7.16). This is fundamentally because the highest value of the heat capacity (4,205 J/kg.K) and the viscosity (0.00152 Pa.s) obtained for the water as compared to the values of these parameters for the milk, which were 3,919 J/kg for the heat capacity and 0.00106 Pa.s for the viscosity.

In [1] the Prandtl number of the tube-side fluid (sea water) at a temperature of 25 °C, in order to cool down a stream of engine oil in a DPHE, was 6.29, with a value for the specific heat capacity and viscosity of 4,004 J/kg.K and 0.000964 Pa.s, respectively. Likewise, in [15] the Prandtl number of

cold water at 27.5 °C, in order to be heated by hot water in a DPHE, was 5.77, with a value for the specific heat capacity and viscosity of 4,179 J/kg.K and 0.000841 Pa.s, respectively.

Regarding the Nusselt number, the tube-side fluid (chilled water) had a value of 1,237.84 for this parameter, which was 12.44 times higher than the value of the Nusselt number (99.49) for the annulus fluid (milk). Considering that the same equation (Prandtl's correlation) was employed to calculate the Nusselt number for both streams, the highest value obtained of this parameter for the chilled water is due to the higher values that the chilled water stream presents for the Reynolds and Prandtl numbers, as compared to the lower values of these parameters for the milk stream. These results agree with those reported by [1], where the Nusselt number of the tube-side fluid (sea water) ranged from 422.0330 - 634.7506, which were higher than the Nusselt number (34.692) of the annulus fluid (engine oil).

Similarly, they also agree with the results reported by [15] where the Nusselt number (375.3) for the tube-side fluid (hot water) is higher than the Nusselt number (89.0) of the annulus fluid (cold fluid). It is worth to mention that all these authors also employed the Prandtl's correlation applied in our study to calculate the Nusselt number for both streams.

The heat transfer coefficient (26,531.78 W/m<sup>2</sup>.K) for the water (tube-side fluid) was 22.57 times higher than the value of the heat transfer coefficient (1,175.24 W/m<sup>2</sup>.K) for the milk (annulus fluid). This result is directly influenced by the higher value of the Nusselt number that the chilled water presents with respect to the value of the Nusselt number for the milk.

These findings coincide with the reported by [1], where the values of the heat transfer coefficients for the tube-side fluid (sea water) ranged between 12,885 – 19,379 W/m<sup>2</sup>.K and were higher than the value of the heat transfer coefficient (64.549 W/m<sup>2</sup>.K) for the annulus fluid (engine oil). In the same way, our results are similar with those reported by [15] where the heat transfer coefficient (4,911 W/m<sup>2</sup>.K) of the tube-side fluid (hot water) is 3.65 times higher than the heat transfer coefficient for the annulus (1,345 W/m<sup>2</sup>.K).

The value of the pressure drop of the tube side fluid (chilled water) is quite high (9,481,246 Pa), and is well above the maximum allowable limit set by the heat transfer system (85,000 Pa). This occurs fundamentally because the high value of the velocity obtained for this fluid (16.64 m/s) and the relatively high number of hairpins (21). This high value of the pressure drop for the chilled water influences on the significant value of the pumping power obtained for this fluid (110.5 kW). On the other hand, the calculated pressure drop for the annulus fluid (milk, 77,392 Pa) is below the maximum allowable set by the system, thus requiring a pumping power of 114.58 W.

As noted in [15], when a significant volume of fluid moves through the tube or the annulus of a DPHE, the pressure drop can exceed the acceptable levels due to high flow velocities, which applies to our research. In these situations, it is advisable to divide the mass flow into several parallel streams, while the lower mass flow rate side can be placed in a series configuration. Consequently, the system will be organized in a parallel-series layout.

Similarly, [14] points out that an increase in fluid velocity results in greater pressure drops, and if the heat exchanger must be integrated into an existing process, the designer should comply with the maximum permissible pressure drop for both streams. This reference also notes that if the calculated pressure drop is too high, it will be necessary to enlarge the flow area, either by increasing the diameter of the tubes or by adding more parallel branches. Conversely, if the determined pressure drop is smaller than allowable, reducing the flow area could be an option. In either scenario, the design process needs to be restarted.

This author further emphasizes that a smaller flow area for both fluids (and subsequently, a reduced tube diameter) leads to increased velocity and heat transfer coefficients, but it also causes greater pressure drops. He recommends, as an initial step, to choose the tube diameter based on fluid velocities, suggesting speeds of 1-2 m/s for liquids with low viscosity, and also proposing that upon the final length is known, the pressure drop for each fluid can be computed, which may demand adjustments to the chosen pipe diameters.

In [15] the pressure drop of the tube-side fluid is 460.1 Pa, thus requiring a pumping power of 0.84 W, while the pressure drop of the annulus fluid is 2,876.4 Pa, therefore needing a pumping power of 5.0 W. In [1], the pressure drop and the pumping power of the tube-side fluid (sea water) for the unfinned clean DPHE design type are 9,376.4 kPa and 27.468 kW, respectively, while the values of these parameters for the unfinned fouled DPHE design type are 9,597 kPa and 28.114 kW, respectively. This reference also reports that the pressure drop and pumping power for the annulus fluid (engine oil) for the unfinned clean DPHE design type are 42.237 MPa and 298.193 kW, respectively, while the values of these parameters for the unfinned fouled DPHE design type are 43.231 MPa and 305.211 kW, respectively.

Lastly, the designed DPHE will cost around USD \$ 45,600 referred to May 2025.

## 5. Conclusions.

In this paper, an unfinned double-pipe heat exchanger was designed from the thermo-hydraulic point of view, to carry out the cooling of a liquid cow's milk stream using chilled water as coolant.

The hot fluid (milk) was located in the annulus, while the cold fluid (chilled water) was located in the inner tube.

Several design; geometrical and operating parameters were calculated for the DPHE such as the heat transfer surface area (12.92 m<sup>2</sup>), total number of hairpins (21), cleanliness factor (0.752) and percent over surface (32.96%), which can be considered as acceptable and adequate. A high value of the required mass flowrate of chilled water was obtained, amounting 9.32 kg/s.

Likewise, the pressure drop of the tube-side fluid is quite high (9,481,246 Pa) and surpasses the maximum allowable pressure drop set by the heat exchange process for both streams (85,000 Pa), whereas the pressure drop of the annulus fluid (77,392 Pa) is below this maximum allowable limit. The high value obtained for the pressure drop of the tube-side fluid increases the required pumping power for this fluid to a significant value (110.5 kW), while the required value of the pumping power for the annulus fluid is 114.58 W. It's concluded that the DPHE designed in this study cannot be successfully implemented in this heat exchange system because of the high values of pressure drop and pumping power obtained for the tube-side fluid (chilled water). The designed DPHE will cost around USD \$ 45,600 based on May 2025. It's recommended to increase the diameter of both pipes and redesign the unfinned DPHE to decrease the pressure drop of the tube-side fluid to a value below the minimum allowable limit set by the heat transfer system for this parameter.

## 6.- Author Contributions (Contributor Roles Taxonomy (CRediT))

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4. Acquisition of funds: Not applicable.
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6. Methodology: Amaury Pérez Sánchez, Laura de la Caridad Arias Aguila, Lizthalía Jiménez Guerra.
7. Project management: Not applicable.
8. Resources: Not applicable.
9. Software: Not applicable.
10. Supervision: Amaury Pérez Sánchez, Laura de la Caridad Arias Aguila.
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14. Writing - revision and editing: Amaury Pérez Sánchez, Laura de la Caridad Arias Aguila.

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### Nomenclature.

$A_o$	Heat transfer surface area	$\text{m}^2$
$A_{ca}$	Net free flow area of the annulus	$\text{m}^2$
$A_{ct}$	Net free flow area of the inner tube	$\text{m}^2$
$A_{hp}$	Heat transfer surface area	$\text{m}^2$
$C_p$	Heat capacity	$\text{kJ/kg.K}$
$CF$	Cleanliness factor	-
$d_e$	External diameter inner tube	$\text{m}$
$d_i$	Internal diameter inner tube	$\text{m}$
$D_e$	Equivalent diameter for heat transfer	$\text{m}$
$D_h$	Hydraulic diameter	$\text{m}$
$D_i$	Internal diameter annulus	$\text{m}$
$f$	Fanning friction factor	-
$f_c$	Corrected Fanning friction factor	-
$h$	Heat transfer coefficient	$\text{W}/\text{m}^2.\text{K}$
$k$	Thermal conductivity	$\text{W}/\text{m.K}$
$k_m$	Thermal conductivity metallic material of the inner pipe	$\text{W}/\text{m.K}$
$L_t$	Tube length	$\text{m}$
$m$	Mass flowrate	$\text{kg}/\text{h}$
$m$	Factor	-
$N_h$	Number of hairpins	-
$Nu$	Nusselt number	-
$OS$	Percent over surface	%
$P$	Pumping power	$\text{kW}$ or $\text{W}$
$Pr$	Prandtl number	-
$\Delta p$	Frictional pressure drop	$\text{Pa}$
$\Delta P_m$	Maximum allowable pressure drop	$\text{Pa}$
$Q$	Heat load	$\text{kW}$
$R$	Fouling factor	$\text{m}^2.\text{K}/\text{W}$
$Re$	Reynolds number	-
$R_{ft}$	Total fouling	$\text{m}^2.\text{K}/\text{W}$
$t$	Temperature cold fluid	$^{\circ}\text{C}$
$\bar{t}$	Average temperature cold fluid	$^{\circ}\text{C}$
$T$	Temperature hot fluid	$^{\circ}\text{C}$
$T_w$	Tube wall temperature	$^{\circ}\text{C}$
$\bar{T}$	Average temperature hot fluid	$^{\circ}\text{C}$
$\Delta T_m$	Log-mean temperature difference	$^{\circ}\text{C}$
$U_c$	Clean overall heat transfer coefficient	$\text{W}/\text{m}^2.\text{K}$
$U_f$	Fouled overall heat transfer coefficient	$\text{W}/\text{m}^2.\text{K}$
$v$	Velocity	$\text{m}/\text{s}$

### Greek symbols

$\rho$	Density	kg/m <sup>3</sup>
$\mu$	Viscosity	Pa.s
$\mu_w$	Viscosity of the fluid at the tube wall temperature	Pa.s
$\eta_p$	Isentropic efficiency of the pump	-

### Subscripts

1	Inlet
2	Outlet
<i>a</i>	Annulus fluid
<i>c</i>	Cold fluid
<i>h</i>	Hot fluid
<i>t</i>	Tube side fluid